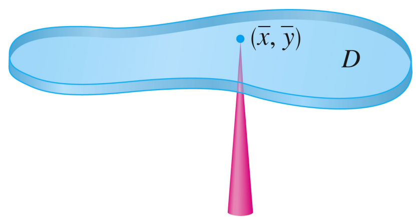
Lesson 33. Application: Center of Mass

1 Definitions

- Suppose we have a **lamina** or thin plate that occupies a region D of the xy-plane



- $\rho(x, y)$ = **density** of the plate at point (x, y) (units: mass per unit area)
- The **mass** of the lamina is given by

$$m = \iint_D \rho(x, y)\, dA$$

- The **moment of the lamina about the x-axis** is

$$M_x = \iint_D y\rho(x, y)\, dA$$

- The **moment of the lamina about the y-axis** is

$$M_y = \iint_D x\rho(x, y)\, dA$$

- The **center of mass** of the lamina is $(\bar{x}, \bar{y})$, where

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m} \iint_D x\rho(x, y)\, dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m} \iint_D y\rho(x, y)\, dA$$

  - The lamina behaves as if the entire mass is concentrated at its center of mass

2 Examples

Example 1. Find the mass and center of mass of a triangular lamina with vertices $(0,0)$, $(1,0)$, and $(0,1)$ if the density function is $\rho(x,y) = x + y$. Just set up the integrals, do not evaluate.

Example 2. Find the mass and center of mass of a lamina that is bounded by the parabolas $y = x^2$ and $x = y^2$ if the density function is $\rho(x,y) = \sqrt{x}$. Just set up the integrals, do not evaluate.

Example 3. The boundary of a lamina consists of the semicircle $y = \sqrt{4 - x^2}$ together with the x-axis. Find the center of mass of the lamina if the density at any point is proportional to its distance from the origin. Use polar coordinates. Just set up the integrals, do not evaluate.

Example 4. A lamina occupies the part of the disk $x^2 + y^2 \leq 1$ in the second quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis. Just set up the integrals, do not evaluate.